

Solving elliptic equations via multiple sums

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Summary.

General or temporal equation of Schrödinger is a mathematic expression of dual trait of corpuscular-wave nature of microparticles of matter and plays fundamental part in nonrelativistic quantum mechanics [1].

If amount of potential energy of field is constant, mathematical dependence between quantitative characteristics of vector field does not contain derivative of time, and dynamic model of Schrödinger becomes stationary. Such equation is used, for example, while studying process of quantization of energy of harmonic oscillator, rotator with free axis, in spectral theory of atoms while studying motion of electrons in coulomb field of core, etc. [2].

The presented work studies elliptic equation that is a modification of stationary equation of Schrödinger. It proves existence and singularity of solution to the problem of Dirichlet in circle for linear differential equation of the second order with special point in the center of research area. The main result of the work is construction of special functions – multiple multinomials of triangle sort, used in calculating coefficients of a line that represents solution. Bibliographic list – 4 items.

Key words: equation, problem, solution, formula, line, multinomial.

Setting the problem.

Let us study stationary equation of Schrödinger with two independent variables that can be formally put in the following expression without physical sense of arguments [3]:

$$u_{xx} + u_{yy} = f(x^2 + y^2) u.$$

In certain cases while $f = -(x^2 + y^2)^l$, $l > 0$ correct settings of edge problems in certain conditions are found for the mentioned equation [4 – 5].

The presented work studies:

Problem 1: In area $D = \{ (x, y): x^2 + y^2 < R^2 \}$ should be found a solution to equation

$$u_{xx} + u_{yy} - \frac{x^2 + y^2}{R^2 + x^2 + y^2} u = 0, \quad (1)$$

that will satisfy border condition

$$u|_{\partial D} = h, \quad h \in C^2(\partial D). \quad (2)$$

Notice 1. Further we shall consider circle radius as a unit of scale of the studied system of coordinates $R=1$ in order to simplify calculations.

Dividing variables according to method of Fourier.

Unconventional solution of border problem 1 will be located in polar coordinates as

$$u(\rho, \varphi) = F(\varphi) \Psi(\rho). \quad (3)$$

As a result of placing product (3) into equation (1) and dividing variables with constant λ we receive equation for the function $\Psi(\rho)$

$$\rho^2 \Psi'' + \rho \Psi' - \left(\frac{\rho^4}{1+\rho^2} + \lambda \right) \Psi = 0, \quad (4)$$

and problem for proper values for the function $F(\varphi)$

$$\left. \begin{aligned} F' + \lambda F &= 0, \\ F(\varphi) &= F(\varphi + 2\pi). \end{aligned} \right\} \quad (5)$$

General solution of homogeneous linear equation (5) is defined via characteristic equation, presented as superposition of harmonics

$$F(\varphi) = A \cos(\sqrt{\lambda} \varphi) + B \sin(\sqrt{\lambda} \varphi).$$

If F is to be single-valued periodic function, the following conditions must be satisfied:

$$\begin{aligned} & A \cos(\sqrt{\lambda}(\varphi + 2\pi)) + B \sin(\sqrt{\lambda}(\varphi + 2\pi)) = \\ & = A \cos(\sqrt{\lambda} \varphi + 2\pi\sqrt{\lambda}) + B \sin(\sqrt{\lambda} \varphi + 2\pi\sqrt{\lambda}). \end{aligned}$$

Selecting proper values of $\lambda = \lambda_n = n^2$, we receive

$$F_n(\varphi) = A_n \cos(n\varphi) + B_n \sin(n\varphi), \quad n = 0; 1; 2; \dots \quad (6)$$

For every fixed value of n of (4) we receive

$$\rho^2 \Psi_n'' + \rho \Psi_n' - \left(\frac{\rho^4}{1+\rho^2} + n^2 \right) \Psi_n = 0. \quad (7)$$

Since equation (7) for each given $n = 0; 1; 2; \dots$ has a special point while $\rho = 0$, solution of it will be presented as a degree line that starts with ρ^{ε_n} [6]:

$$\Psi_n = \rho^{\varepsilon_n} \sum_{i=0}^{\infty} c_{i,n} \rho^i = \rho^{\varepsilon_n} (c_{0,n} + c_{1,n} \rho + c_{i,n} \rho^2 + \dots + c_{i,n} \rho^i + \dots). \quad (8)$$

Values of characteristic index ε_n and coefficients of $c_{i,n}$ can be defined via placing line (8) into equation (7). As we consequently equalize coefficients by $x^\varepsilon, x^{\varepsilon+1}, x^{\varepsilon+2}, \dots$ to zero, we receive a system of equations:

$$\left. \begin{aligned}
(\varepsilon^2 - n^2)c_0 &= 0, \\
[(\varepsilon + 1)^2 - n^2]c_1 &= 0, \\
[(\varepsilon + 2)^2 - n^2]c_2 &= 0, \\
[(\varepsilon + 3)^2 - n^2]c_3 &= 0, \\
[(\varepsilon + 4)^2 - n^2]c_4 &= c_0, \\
[(\varepsilon + 5)^2 - n^2]c_5 &= c_1, \\
[(\varepsilon + 6)^2 - n^2]c_6 &= c_2 - c_0, \\
[(\varepsilon + 7)^2 - n^2]c_7 &= c_3 - c_1, \\
[(\varepsilon + 8)^2 - n^2]c_8 &= c_4 - c_2 + c_0, \\
[(\varepsilon + 9)^2 - n^2]c_9 &= c_5 - c_3 + c_1, \\
[(\varepsilon + 10)^2 - n^2]c_{10} &= c_6 - c_4 + c_2 - c_0, \\
[(\varepsilon + 11)^2 - n^2]c_{11} &= c_7 - c_5 + c_3 - c_1, \\
[(\varepsilon + 12)^2 - n^2]c_{12} &= c_8 - c_6 + c_4 - c_2 + c_0, \\
[(\varepsilon + 13)^2 - n^2]c_{13} &= c_9 - c_7 + c_5 - c_3 + c_1, \\
\vdots &
\end{aligned} \right\}$$

Considering $c_0 \neq 0$, from the first equation we find $\varepsilon = \pm n$.

In order to define singular border while $\rho \rightarrow 0$ we consider solution of equation (7) as $\varepsilon_n = n$, $n = 0; 1; 2; \dots$. Then, from the last system we conclude $c_{1,n} = c_{2,n} = c_{3,n} = 0$. In this case all further odd coefficients of $c_{2i+1,n}$, $i = 2; 3; \dots$ must also equal zero, and all even coefficients are defined through the sum of previous ones according to alternative formulas

$$\left. \begin{aligned}
c_{2i,n} &= \frac{c_{2i-4,n} - 4(i-1)(i-1+n)c_{2i-2,n}}{4i(i+n)}, \quad i = 2; 3; \dots, \\
c_{2i,n} &= \sum_{j=0}^{i-2} \frac{(-1)^{i+j} c_{2j,n}}{4i(i+n)}, \quad i = 2; 3; \dots, \\
c_{4+2i,n} &= \sum_{j=0}^i \frac{(-1)^{i+j} c_{2j,n}}{4(2+i)(2+i+n)}, \quad i = 0; 1; \dots
\end{aligned} \right\} \quad (9)$$

Consequent implementation of formula (9) while $i = 0; 1; 2; 3; \dots$ allows us to receive expression $c_{4i,n}$ through $c_{0,n}$:

$$\begin{aligned}
c_{4,n} &= \frac{c_{0,n}}{4 \cdot 2(2+n)}, \quad c_{6,n} = \frac{-c_{0,n}}{4 \cdot 3(3+n)}, \quad c_{8,n} = \frac{c_{0,n}}{4 \cdot 4(4+n)} \left[1 + \frac{1}{4 \cdot 2(2+n)} \right], \\
c_{10,n} &= \frac{-c_{0,n}}{4 \cdot 5(5+n)} \left[1 + \frac{1}{4 \cdot 2(2+n)} + \frac{1}{4 \cdot 3(3+n)} \right], \\
c_{12,n} &= \frac{c_{0,n}}{4 \cdot 6(6+n)} \left[1 + \frac{1}{4 \cdot 2(2+n)} + \frac{1}{4 \cdot 3(3+n)} + \frac{1}{4 \cdot 4(4+n)} + \frac{1}{4^2 \cdot 2 \cdot 4(2+n)(4+n)} \right], \\
c_{14,n} &= \frac{-c_{0,n}}{4 \cdot 7(7+n)} \left[1 + \frac{1}{4 \cdot 2(2+n)} + \frac{1}{4 \cdot 3(3+n)} + \frac{1}{4 \cdot 4(4+n)} + \frac{1}{4 \cdot 5(5+n)} + \frac{1}{4^2 \cdot 2 \cdot 4(2+n)(4+n)} + \right. \\
&\quad \left. + \frac{1}{4^2 \cdot 2 \cdot 5(2+n)(5+n)} + \frac{1}{4^2 \cdot 3 \cdot 5(3+n)(5+n)} \right],
\end{aligned}$$

$$c_{16,n} = \frac{-c_{0,n}}{4 \cdot 8 (8+n)} \left[1 + \frac{1}{4 \cdot 2 (2+n)} + \frac{1}{4 \cdot 3 (3+n)} + \frac{1}{4 \cdot 4 (4+n)} + \frac{1}{4 \cdot 5 (5+n)} + \frac{1}{4 \cdot 6 (6+n)} + \right. \\ \left. + \frac{1}{4^2 \cdot 2 \cdot 4 (2+n) (4+n)} + \frac{1}{4^2 \cdot 2 \cdot 5 (2+n) (5+n)} + \frac{1}{4^2 \cdot 2 \cdot 6 (2+n) (6+n)} + \frac{1}{4^2 \cdot 3 \cdot 5 (3+n) (5+n)} + \right. \\ \left. + \frac{1}{4^2 \cdot 3 \cdot 6 (3+n) (6+n)} + \frac{1}{4^2 \cdot 4 \cdot 6 (4+n) (6+n)} + \frac{1}{4^3 \cdot 2 \cdot 4 \cdot 6 (2+n) (4+n) (6+n)} \right] \\ \dots$$

While $i = 2; 3; 4; \dots$, $\alpha_i = \begin{cases} i, & \text{if } i \text{ is even,} \\ i-1, & \text{if } i \text{ is odd,} \end{cases}$ or $\alpha_i = i + \frac{(-1)^{i-1}}{2}$ $c_{4+2i,n} =$

$$\frac{(-1)^i c_{0,n}}{4 (i+2) (i+2+n)} \left[1 + \frac{1}{4 \cdot 2 (2+n)} + \frac{1}{4 \cdot 3 (3+n)} + \frac{1}{4 \cdot 4 (4+n)} + \right. \\ \left. + \frac{1}{4 \cdot 5 (5+n)} + \frac{1}{4 \cdot 6 (6+n)} + \frac{1}{4^2 \cdot 2 \cdot 4 (2+n) (4+n)} + \frac{1}{4^2 \cdot 2 \cdot 5 (2+n) (5+n)} + \right. \\ \left. + \frac{1}{4^2 \cdot 2 \cdot 6 (2+n) (6+n)} + \frac{1}{4^2 \cdot 3 \cdot 5 (3+n) (5+n)} + \frac{1}{4^2 \cdot 3 \cdot 6 (3+n) (6+n)} + \right. \\ \left. + \frac{1}{4^2 \cdot 4 \cdot 6 (4+n) (6+n)} + \frac{1}{4^3 \cdot 2 \cdot 4 \cdot 6 (2+n) (4+n) (6+n)} + \dots + \right. \\ \left. + \frac{1}{4^{\alpha_i/2} i (i-2) (i-4) \dots \alpha_i (i+n) (i-2+n) (i-4+n) \dots (\alpha_i+n)} \right]. \quad (10)$$

Let us designate special auxiliary functions

$$P_{i,l} = \sum_{\tau_1=0}^{i-2-2l} \sum_{\tau_2=0}^{\tau_1} \dots \sum_{\tau_{l+1}=0}^{\tau_l} \prod_{\eta=0}^l \frac{1}{(2+2\eta+\tau_{l+1-\eta})(2+2\eta+\tau_{l+1-\eta}+n)}. \quad (11)$$

Considering $c_{0,n} = 4$, equations of system (10) while $i \geq 2$ can be expressed as

$$c_{8,n} = \frac{1}{4(4+n)} \left[1 + \frac{1}{2^2} P_{2,0} \right], \quad c_{10,n} = \frac{-1}{5(5+n)} \left[1 + \frac{1}{2^2} P_{3,0} \right], \\ c_{12,n} = \frac{1}{6(6+n)} \left[1 + \frac{1}{2^2} P_{4,0} + \frac{1}{2^4} P_{4,1} \right], \quad c_{14,n} = \frac{-1}{7(7+n)} \left[1 + \frac{1}{2^2} P_{5,0} + \frac{1}{2^4} P_{5,1} \right], \\ c_{16,n} = \frac{1}{8(8+n)} \left[1 + \frac{1}{2^2} P_{6,0} + \frac{1}{2^4} P_{6,1} + \frac{1}{2^6} P_{6,2} \right], \quad c_{18,n} = \frac{-1}{9(9+n)} \left[1 + \frac{1}{2^2} P_{7,0} + \frac{1}{2^4} P_{7,1} + \frac{1}{2^6} P_{7,2} \right] \\ \dots \\ c_{4+2i,n} = \frac{(-1)^i}{(i+2)(i+2+n)} \left[1 + \sum_{l=0}^{\frac{\alpha_i-2}{2}} \frac{1}{2^{\alpha(2+2l)}} P_{i,l} \right]. \quad (12)$$

Example of calculating coefficients of line (8).

According to formulas (11) - (12) we define coefficients of line (8): a) $c_{12,n}$ and b) $c_{14,n}$.

Solution. a) $c_{12,n} = c_{(4+2 \cdot 4),n} = \frac{(-1)^4}{(4+2)(4+2+n)} \left[1 + \sum_{l=0}^{\frac{\alpha_4-2}{2}} \frac{1}{2^{\alpha(2+2l)}} P_{4,l} \right] =$

$$\frac{1}{6(6+n)} \left[1 + \sum_{l=0}^1 \frac{1}{2^{\alpha(2+2l)}} P_{4,l} \right] = \frac{1}{6(6+n)} \left[1 + \frac{1}{2^2} P_{4,0} + \frac{1}{2^4} P_{4,1} \right].$$

Let us define value of coefficient $P_{4,0}$:

$$P_{4,0} = \sum_{\tau_1=0}^2 \prod_{\eta=0}^0 \frac{1}{(2+2\eta+\tau_{1-\eta})(2+2\eta+\tau_{1-\eta}+n)} = \sum_{\tau_1=0}^2 \frac{1}{(2+\tau_1)(2+\tau_1+n)} = >$$

$$P_{4,0} = \frac{1}{2(2+n)} + \frac{1}{3(3+n)} + \frac{1}{4(4+n)}$$

We define $P_{4,1} = \sum_{\tau_1=0}^0 \sum_{\tau_2=0}^{\tau_1} \prod_{\eta=0}^1 \frac{1}{(2+2\eta+\tau_{2-\eta})(2+2\eta+\tau_{2-\eta}+n)}$. Since

$$\prod_{\eta=0}^1 \frac{1}{(2+2\eta+\tau_{2-\eta})(2+2\eta+\tau_{2-\eta}+n)} = \frac{1}{(2+\tau_2)(2+\tau_2+n)(2+2+\tau_{2-1})(2+2+\tau_{2-1}+n)} =$$

$$= \frac{1}{(2+\tau_2)(2+\tau_2+n)(4+\tau_1)(4+\tau_1+n)}, \text{ we receive}$$

$$P_{4,1} = \sum_{\tau_1=0}^0 \sum_{\tau_2=0}^{\tau_1} \frac{1}{(2+\tau_2)(2+\tau_2+n)(4+\tau_1)(4+\tau_1+n)} =$$

$$= \sum_{\tau_2=0}^0 \frac{1}{(2+\tau_2)(2+\tau_2+n)4(4+n)} = \frac{1}{2(2+n)4(4+n)}.$$

Placing $P_{4,0}$ and $P_{4,1}$ into formula $c_{12,n} = \frac{1}{6(6+n)} \left[1 + \frac{1}{2^2} P_{4,0} + \frac{1}{2^4} P_{4,1} \right]$, we receive

$$c_{12,n} = \frac{1}{6(6+n)} \left[1 + \frac{1}{4 \cdot 2(2+n)} + \frac{1}{4 \cdot 3(3+n)} + \frac{1}{4 \cdot 4(4+n)} + \frac{1}{4^2 \cdot 2 \cdot 4(2+n)(4+n)} \right].$$

$$c) \ c_{14,n} = c_{(4+2 \cdot 5),n} = \frac{(-1)^5}{(5+2)(5+2+n)} \left[1 + \sum_{l=0}^{\frac{\alpha_5-2}{2}} \frac{1}{2^{\alpha(2+2 \cdot l)}} P_{5,l} \right] = \frac{-1}{7(7+n)} \left[1 + \sum_{l=0}^1 \frac{1}{2^{\alpha(2+2 \cdot l)}} P_{5,l} \right] =$$

$$= \frac{-1}{7(7+n)} \left[1 + \frac{1}{2^{\alpha_2}} P_{5,0} + \frac{1}{2^{\alpha_4}} P_{5,1} \right] = \frac{-1}{7(7+n)} \left[1 + \frac{1}{2^2} P_{5,0} + \frac{1}{2^4} P_{5,1} \right],$$

$$P_{5,0} = \sum_{\tau_1=0}^3 \prod_{\eta=0}^0 \frac{1}{(2+2\eta+\tau_{1-\eta})(2+2\eta+\tau_{1-\eta}+n)} = \sum_{\tau_1=0}^3 \frac{1}{(2+\tau_1)(2+\tau_1+n)} =$$

$$= \frac{1}{2(2+n)} + \frac{1}{3(3+n)} + \frac{1}{4(4+n)} + \frac{1}{5(5+n)},$$

$$P_{5,1} = \sum_{\tau_1=0}^1 \sum_{\tau_2=0}^{\tau_1} \prod_{\eta=0}^1 \frac{1}{(2+2\eta+\tau_{2-\eta})(2+2\eta+\tau_{2-\eta}+n)}.$$

$$\prod_{\eta=0}^1 \frac{1}{(2+2\eta+\tau_{2-\eta})(2+2\eta+\tau_{2-\eta}+n)} = \frac{1}{(2+\tau_2)(2+\tau_2+n)(4+\tau_1)(4+\tau_1+n)},$$

$$\begin{aligned}
P_{5,1} &= \sum_{\tau_1=0}^1 \sum_{\tau_2=0}^{\tau_1} \frac{1}{(2+\tau_2)(2+\tau_2+n)(4+\tau_1)(4+\tau_1+n)} = \\
&= \sum_{\tau_2=0}^0 \frac{1}{(2+\tau_2)(2+\tau_2+n)4(4+n)} + \sum_{\tau_2=0}^1 \frac{1}{(2+\tau_2)(2+\tau_2+n)(4+1)(4+1+n)} = \\
&= \frac{1}{2(2+n)4(4+n)} + \frac{1}{2(2+n)5(5+n)} + \frac{1}{3(3+n)5(5+n)}.
\end{aligned}$$

$$c_{14,n} = \frac{-1}{7(7+n)} \left[1 + \frac{1}{2^2} P_{5,0} + \frac{1}{2^4} P_{5,1} \right] \Rightarrow$$

$$\begin{aligned}
c_{14,n} &= \frac{-1}{7(7+n)} \left[1 + \frac{1}{4 \cdot 2(2+n)} + \frac{1}{4 \cdot 3(3+n)} + \frac{1}{4 \cdot 4(4+n)} + \frac{1}{4 \cdot 5(5+n)} + \right. \\
&\quad \left. + \frac{1}{4^2 \cdot 2 \cdot 4(2+n)(4+n)} + \frac{1}{4^2 \cdot 2 \cdot 5(2+n)(5+n)} + \frac{1}{4^2 \cdot 3 \cdot 5(3+n)(5+n)} \right].
\end{aligned}$$

Algorithm of calculating coefficients P_{ij} .

In order to simplify the process, algorithm of calculating coefficients of line

$$P_{i,l} = \sum_{\tau_1=0}^{i-2-2l} \sum_{\tau_2=0}^{\tau_1} \dots \sum_{\tau_{l+1}=0}^{\tau_l} \prod_{\eta=0}^l \frac{1}{(2+2\eta+\tau_{l+1-\eta})(2+2\eta+\tau_{l+1-\eta}+n)}, \quad i \geq 2+2l.$$

will be studied at the example of forming multiple sums, presented as

$$Q_{i,l} = \sum_{\tau_1=0}^{i-2-2l} \sum_{\tau_2=0}^{\tau_1} \dots \sum_{\tau_{l+1}=0}^{\tau_l} \prod_{\eta=0}^l (2+2\eta+\tau_{l+1-\eta}), \quad i \geq 2+2l,$$

Let us first define consequences of sums $Q_{i,l}$ with the same index of l .

While $l = 1$ we receive

$$\begin{aligned}
Q_{4,1} &= \sum_{\tau_1=0}^0 \sum_{\tau_2=0}^{\tau_1} \prod_{\eta=0}^1 (2+2\eta+\tau_{2-\eta}) = \sum_{\tau_1=0}^0 \sum_{\tau_2=0}^{\tau_1} (2+\tau_2)4 = \sum_{\tau_2=0}^0 (2+\tau_2)4 = 2 \cdot 4, \\
Q_{5,1} &= \sum_{\tau_1=0}^1 \sum_{\tau_2=0}^{\tau_1} \prod_{\eta=0}^1 (2+2\eta+\tau_{2-\eta}) = \sum_{\tau_1=0}^1 \sum_{\tau_2=0}^{\tau_1} (2+\tau_2)(2+2+\tau_1) = \\
&= \sum_{\tau_2=0}^0 (2+\tau_2)(2+2) + \sum_{\tau_2=0}^1 (2+\tau_2)(2+2+1) = 2 \cdot 4 + 2 \cdot 5 + 3 \cdot 5, \\
Q_{6,1} &= \sum_{\tau_1=0}^2 \sum_{\tau_2=0}^{\tau_1} \prod_{\eta=0}^1 (2+2\eta+\tau_{2-\eta}) = \sum_{\tau_1=0}^2 \sum_{\tau_2=0}^{\tau_1} (2+\tau_2)(2+2+\tau_1) = \\
&= \sum_{\tau_2=0}^0 (2+\tau_2)(2+2) + \sum_{\tau_2=0}^1 (2+\tau_2)(2+2+1) + \sum_{\tau_2=0}^2 (2+\tau_2)(2+2+2) =
\end{aligned}$$

$$= 2 \cdot 4 + 2 \cdot 5 + 3 \cdot 5 + 2 \cdot 6 + 3 \cdot 6 + 4 \cdot 6,$$

$$Q_{71} = \sum_{\tau_1=0}^3 \sum_{\tau_2=0}^{\tau_1} \prod_{\eta=0}^1 (2 + 2\eta + \tau_{2-\eta}) = \sum_{\tau_1=0}^3 \sum_{\tau_2=0}^{\tau_1} (2 + \tau_2)(2 + 2 + \tau_1) =$$

$$= \sum_{\tau_2=0}^0 (2 + \tau_2) \cdot 4 + \sum_{\tau_2=0}^1 (2 + \tau_2) \cdot 5 + \sum_{\tau_2=0}^2 (2 + \tau_2) \cdot 6 + \sum_{\tau_2=0}^3 (2 + \tau_2) \cdot 7 =$$

$$= 2 \cdot 4 + 2 \cdot 5 + 3 \cdot 5 + 2 \cdot 6 + 3 \cdot 6 + 4 \cdot 6 + 2 \cdot 7 + 3 \cdot 7 + 4 \cdot 7 + 5 \cdot 7 \quad \text{etc.}$$

Consequences of multipliers in summands of the studied sums $Q_{5,1}$, $Q_{6,1}$, Q_{71} can be easily composed with triangular matrixes

$$A_{5,1} = \begin{pmatrix} 2 \cdot 4 & 2 \cdot 5 \\ 0 & 3 \cdot 5 \end{pmatrix}, \quad A_{6,1} = \begin{pmatrix} 2 \cdot 4 & 2 \cdot 5 & 2 \cdot 6 \\ 0 & 3 \cdot 5 & 3 \cdot 6 \\ 0 & 0 & 4 \cdot 6 \end{pmatrix}, \quad A_{71} = \begin{pmatrix} 2 \cdot 4 & 2 \cdot 5 & 2 \cdot 6 & 2 \cdot 7 \\ 0 & 3 \cdot 5 & 3 \cdot 6 & 3 \cdot 7 \\ 0 & 0 & 4 \cdot 6 & 4 \cdot 7 \\ 0 & 0 & 0 & 5 \cdot 7 \end{pmatrix},$$

As this trait is possessed by all expressions of $Q_{i,l}$, we shall call them *multiple multinomials of triangular presentation*, and functions $P_{i,l}(n)$ that present their modification, will therefore be named *multiple sums of triangular presentation*.

Solution of edge problem 1. Uniting the received results, we define solution of problem (1) - (2) in polar coordinates

$$\left. \begin{aligned} \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial u}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 u}{\partial \rho^2} - \frac{\rho^2}{1 + \rho^2} u &= 0 \\ u|_{\rho=1} &= h \end{aligned} \right\} \quad (13)$$

According to formula (3): $u(\rho, \varphi) = F(\varphi) \Psi(\rho)$.

Above we have proved that after splitting variables of problem (13) we receive two equations, the first one

$$F'' + \lambda F = 0$$

has proper solutions (6)

$$\Phi_n(\varphi) = A_n \cos(n\varphi) + B_n \sin(n\varphi), \quad n = 0; 1; \dots,$$

For each fixed n the second equation

$$\rho^2 \Psi_n'' + \rho \Psi_n' - \left(\frac{\rho^4}{1 + \rho^2} + n^2 \right) \Psi_n = 0,$$

has proper solutions, presented as (8)

$$\Psi_n = \rho^n \left(4 + \frac{1}{2(2+n)} \rho^4 - \frac{1}{3(3+n)} \rho^6 + \dots + \sum_{i=2}^{\infty} c_{4+2i,n} \rho^{4+2i} \right).$$

Coefficients of degree line (8) are defined according to formulas (12)

$$c_{4+2i,n} = \frac{(-1)^i}{(i+2)(i+2+n)} \left[1 + \sum_{l=0}^{\frac{\alpha_i-2}{2}} \frac{1}{2^{\alpha_{(2+2l)}}} P_{i,l} \right], \quad n = 0; 1; 2; \dots,$$

in which $\alpha_i = i + \frac{(-1)^i - 1}{2}$, and multiple multinomials $P_{i,l}$ are defined by correlations (11)

$$P_{i,l} = \sum_{\tau_1=0}^{i-2-2l} \sum_{\tau_2=0}^{\tau_1} \dots \sum_{\tau_{l+1}=0}^{\tau_l} \prod_{\eta=0}^l \frac{1}{(2+2\eta+\tau_{l+1-\eta})(2+2\eta+\tau_{l+1-\eta}+n)}$$

Placing expressions $\Phi_n(\varphi)$ and $\Psi_n(\rho)$ into formula (3), we define two systems of proper functions $\{\cos(n\varphi) \Psi_n\}_0^\infty$ and $\{\sin(n\varphi) \Psi_n\}_0^\infty$ that are met by certain solutions of the first equations (13)

$$u_n(\varphi, \rho) = \Psi_n(\rho)[A_n \cos(n\varphi) + B_n \sin(n\varphi)].$$

Superposition of all these solutions

$$u(\varphi, \rho) = \sum_{n=0}^{\infty} \Psi_n(\rho)[A_n \cos(n\varphi) + B_n \sin(n\varphi)] \quad (14)$$

Will also be solution of this equation.

Coefficients A_n and B_n are defined from border condition (13)

$$u(\varphi, 1) = h(\varphi) = \sum_{n=0}^{\infty} \Psi_n(1)[A_n \cos(n\varphi) + B_n \sin(n\varphi)], \quad (15)$$

if function $h \in C^2(\partial D)$ is distributed into absolutely and equally convergent trigonometrical line of Fourier

$$h(\varphi) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(n\varphi) + b_n \sin(n\varphi)], \quad (16)$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} h(t) dt, \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} h(t) \cos(n\varphi) dt, \quad n = 1; 2; \dots$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} h(t) \sin(n\varphi) dt, \quad n = 1; 2; \dots$$

Comparing lines (15) and (16), we receive

$$A_0 = \frac{a_0}{2 \Psi_0(1)}, \quad A_n = \frac{a_n}{\Psi_n(1)}, \quad B_n = \frac{b_n}{\Psi_n(1)}, \quad n = 1; 2; \dots \quad (17)$$

Applicability of the principle of superposition.

Convergence of the constructed lines, possibility of their differentiation in circle \overline{D} and also continuity of function $u(\varphi, \rho)$ at the border of this circle are proved via classical methods [2, p. 308 – 310].

Via alternating method of Schwartz the formed solution can be prolonged outside circle borders into areas of more general view [1].

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