Pavlikov Sergey Vladimirovich, Naberezhnye Chelny branch of Kazan State Technical University named after Tupolev, Professor of the Department of natural Sciences
Павликов Сергей Владимирович, Набережночелнинский филиал Казанского Национального Исследовательского Технического Университета им. А.Н. Туполева, Профессор кафедры естественнонаучных дисциплин
Savin Igor Alekseevitch, Naberezhnye Chelny branch of Kazan State Technical University named after Tupolev, Head of Department, Department of Engineering and technology engineering industries
Савин Игорь Алексеевич, Набережночелнинский филиал Казанского Национального Исследовательского Технического Университета им. А.Н. Туполева, заведующий кафедрой Конструирование и технологии машиностроительных производств

About the investigation of the stability of functional differential equations of retarded type

.

In the work there is investigated the stability of the zero solution of a non-autonomous functional differential equation of the delayed type by means of limiting equations and Lyapunov constant-sign functional. An appropriate illustrating example is given.

1. Introduction. Basic definitions and limiting equations.

Suppose R is a real axis, R^n is a real linear space of n-vectors x with a norm |x|, $h = const > 0$ is a real number, C is the Banach space of continuous functions $\varphi : [-h,0] \to R^n$ with a norm $\|\varphi\| = \sup(|\varphi(s)|, -h \le s \le 0)$, C_H is a space $\{\varphi \in C : \|\varphi\| < H, H > 0\}$. For a continuous function $x :]-\infty,+\infty[\to R^n$ and every $t \in R$, the function $x_t(s) \in C_H$ is defined by the equality $x_t(s) = x(t+s), s \in [-h,0]$. A right-hand derivative is denoted by $\dot{x}(t)$.

The functional differential equation with a finite delay

$$\dot{x}(t) = f(t, x_t), f(t,0) \equiv 0 \tag{1}$$

is considered, where $f : R^+ \times C_H \to R^n$ is a continuous function which satisfies the assumptions 1-3 [1, 2].

2. Basic results. Stability theorems.

We will investigate the problem of the stability on the base of Lyapunov constant-sign functionals. We shall use the following definitions.

Definition 1. The solution $x = 0$ of Eq.(1) is stable with respect to set $\Lambda \subset C_H$, if, for any $\varepsilon > 0$ one can get $\delta = \delta(\varepsilon) > 0$, so that for $\varphi \in \Lambda \cap \{\|\varphi\| < \delta\}$ it is true that $|x_t(t,0,\varphi)| < \varepsilon$ for each solution $x(t,0,\varphi)$ of Eq.(1) for any $t \ge 0$.

Definition 2. The solution $x = 0$ of Eq.(1) is uniformly asymptotically stable with respect to set $\Lambda \subset C_H$, if it is stable with respect to $\Lambda \subset C_H$ and a $\Delta > 0$ exists, so that for any $\varepsilon > 0$ one can get $T = T(\varepsilon) > 0,$ so that for every $\varphi \in \Lambda \cap \{\|\varphi\| < \Delta\}$ it is true that $\|x_t(0,\varphi)\| < \varepsilon$ for any $t \ge T$.

Definition 3. The solution $x = 0$ is a point of uniform attraction for the whole family of limiting equations $\{\dot{x}(t) = f^*(t, x_t)\}$ with respect to set $\Lambda \subset C_H$, if a Δ exists, so that for any $\varepsilon > 0$ there is $T = T(\varepsilon) > 0,$ so that for any solution $x^*(t,0,\varphi)$, $\varphi \in \Lambda \cap \{\|\varphi\| < \Delta\}$ of any equation $\dot{x}(t) = f^*(t, x_t)$ for any $t \ge T$ the inequality $\|x_t^*(0,\varphi)\| < \varepsilon$ holds.

Suppose $V : R^+ \times C_H \to R^+$ is a certain continuous functional, $x = x(t,\alpha,\varphi), (\alpha,\varphi) \in R^+ \times C_H$ is a certain solution of Eq.(1). Along this solution the functional V is a continuous time-dependent function $V(t)=V(t,x_t(\alpha,\varphi))$. For this function it is possible to define an upper right-hand derivative $\dot{V}(t,\varphi)$.

Let us denote as $\omega_i(u)$ continuous strictly monotonically increasing functions $\omega_i : R^+ \to R^+, \omega_i(0)=0$.

Definition 4. Let us define a set for the functional $V(t,\varphi)$:

$V^{-1}(\infty,0)=\{\varphi\in C_H : \exists\varphi_n \in C_H, \exists t_n \to +\infty : \varphi_n \to \varphi, V(t_n,\varphi_n) \to 0, n \to +\infty\}$.

The definitions which have been introduced enable us to derive the sufficient conditions of stability and asymptotic stability when a non-negative functional with a non-positive derivative exists.

Theorem 1. Suppose that:

1) a continuous functional $V : R^+ \times C_H \to R^+$ exists, so that $V(t,\varphi) \geq 0, V(t,0) \equiv 0, \dot{V}(t,\varphi) \leq 0, (t,\varphi)\in R^+ \times C_H$;

2) the solution $x=0$ is a point of uniform attraction for solutions $\{\dot{x}(t) = f^*(t,x_t)\}$ with respect to the set $\Lambda_0 = V^{-1}(\infty,0)$.

Then the solution $x=0$ is stable by Lyapunov.

Theorem 2. We will assume that:

1) the continuous functional exists $V : R^+ \times C_H \to R^+$ such that:
$0 \leq V(t,\varphi) \leq \omega(\|\varphi\|), V(t,0) \equiv 0, \dot{V}(t,\varphi) \leq 0,$
$(t,\varphi)\in R^+ \times C_H$
2) the solution x=0 is asymptotically stable uniformly with respect to the set $\Lambda_0 = V^{-1}(\infty,0)$.

Then the solution x=0 of equation (1) is uniformly stable by Lyapunov.

3. Conclusion. There is the development of the method of Lyapunov constant -sign functionals with using of the limit equations in the work. The obtained theorems 1,2 develop and expand some results from [2].

REFERENCES

1. ANDREEV, A.S. and KHUSANOV, D. Kh., Limit equations in the problem of stability of a functional-differential equation. *Defferents. Uravn.*, 1998, **34**, 435 – 440.

2. PAVLIKOV, S.V., Lyapunov constant- sign functional in the problem of stability of a functional-differential equation. *PMM., 2007, 3, 377 – 388.*